## II B.Tech - II Semester - Regular Examinations - JULY 2022

# TRANSFORM TECHNIQUES, NUMERICAL METHODS AND NUMBER THEORY (INFORMATION TECHNOLOGY) 

Duration: 3 hours
Max. Marks: 70
Note: 1. This paper contains questions from 5 units of Syllabus. Each unit carries 14 marks and have an internal choice of Questions.
2. All parts of Question must be answered in one place.

## UNIT - I

1. a) Find the Laplace transform of $\left(\sqrt{t}-\frac{1}{\sqrt{t}}\right)^{3}$
b) Find $L\left\{t e^{-3 t} \cos 2 t\right\}$

OR
2. a) Find the Laplace transform of
(i) $e^{2 t} \sin 2 t$
(ii) $\sin ^{2} 2 t$
7 M
b) Find the Laplace transform of
(i) $t \cos (\omega t+\theta)$
(ii) $e^{-3 t} u(t-2)$
7 M

## UNIT - II

3. 

a) Find (i) $L^{-1}\left\{\frac{s^{2}}{s^{4}-a^{4}}\right\}$
(ii) $L^{-1}\left\{\frac{s}{(s+3)^{2}}\right\}$
b) Evaluate (i) $L^{-1}\left\{\log \frac{s+1}{s-1}\right\}$
(ii) $L^{-1}\left\{\frac{s}{\left(s^{2}-4\right)^{2}}\right\}$ 7 M OR
4.
a) Find (i) $L^{-1}\left\{\frac{s}{2 s^{2}-8}\right\}$
(ii) $L^{-1}\left\{\frac{s+1}{s^{2}+s+1}\right\}$
b) Evaluate $L^{-1}\left\{\frac{s^{2}}{\left(s^{2}+a^{2}\right)\left(s^{2}+b^{2}\right)}\right\}$ using convolution theorem.

## UNIT-III

5. a) Find the root of the equation $2 x-\log _{10} x=7$ which lies between 3.5 and 4 by Regula falsi method.
b) Apply Lagrange's method to find the value of $y$ for $x=10$ from the following table:

| $x$ | 5 | 6 | 9 | 11 |
| :--- | :--- | :--- | :--- | :--- |
| $y$ | 12 | 13 | 14 | 16 |

6. a) Find a real root of the equation $x^{3}-5 x-7=0$ using the Newton-Raphson method.
b) Estimate the values of $f(42)$ from the following data.

| $x$ | 20 | 25 | 30 | 35 | 40 | 45 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{y}=\mathrm{f}(\mathrm{x})$ | 354 | 332 | 291 | 260 | 231 | 204 |

## UNIT - IV

7. a) Find the value of $y$ for $x=0.4$ by Picard's method, $y^{\prime}=x^{2}+y^{2}, y(0)=0$.
b) By modified Euler's method, find $y(0.1), y(0.2)$ and given that $\frac{d y}{d x}=x+y, y(0)=1$
8. a) Using Taylor 's series method find an approximate value of $y$ at $x=0.2$ for the differential equation $y^{\prime}-2 y=3 e^{x}, y(0)=0$. Compare the numerical solution obtained with the exact solution.
b) Find $y(0.1)$ using Runge Kutta fourth order formula, 7 M
given that $\frac{d y}{d x}=x^{2}-y, y(0)=1$ and compare the result with analytical method.

## UNIT - V

9. a) Define greatest common divisor (GCD) of two integers a and b . Also find GCD of 24 and 96 . 7 M
b) Find all positive integers $n$ for which the congruence $a^{25} \equiv a \bmod n$ holds for all integers $a$. 7 M

OR
10. a) By Fermat's theorem, Find the remainder when $5^{38}$ is
divided by 11 .
b) Show that if for a positive integer $n$ the number $2^{n}+1$ is prime then $n$ must be a power of 2 . 7 M

